# Finite Math - J-term 2019

Lecture Notes - 1/10/2019

## Homework

- Section 3.2 - 73, 74
- Section 3.3 - 7, 9, 11, 15, 17, 27, 31, 33, 39
- Section 3.4 - 7, 15, 18, 27, 30

## Section 3.2 - Compound and Continuous Compound Interest

**Example 1.** *The Russell Index tracks the average performance of various groups of stocks. On average, a* $10,000 *investment in mid-cap growth funds over a* 10*-year period would have grown to* $63,000*. (A mid-cap fund is a type of stock fund that invests in mid-sized companies. See Investopedia for more information.) What annual nominal rate would produce the same growth if interest were compounded (a) annually, (b) continuously. Express answers as a percentage, rounded to three decimal places.*

**Solution.**

**Example 2.** *A promissory note will pay* $\$50,000$ *at maturity* $6$ *years from now. If you pay* $\$28,000$ *for the note now, what rate would you earn if interest were compounded (a) quarterly, (b) continuously?*

**Solution.**

# Section 3.3 - Future Value of an Annuity; Sinking Funds

**Annuities.** At this point, we have only discussed investments where there was one initial deposit and a final payoff. But what if you make regular equal payments into an account? An *annuity* is a sequence of equal periodic payments. If payments are made at the end of each time interval, then the annuity if called an *ordinary annuity*. Our goal in this section will be to find the future value of an annuity.

**Example 3.** *Suppose you decide to deposit* $\$100$ *every* $6$ *months into a savings account which pays* $6\%$ *compounded semiannually. If you make* $6$ *deposits, one at the end of each interest payment period over the course of* $3$ *years, how much money will be in the account after the last deposit is made?*

**Solution.**

This gives rise to the following formula

**Definition 1** (Future Value of an Ordinary Annuity).

*where*

$$\begin{aligned} FV &= \textit{future value} \\ PMT &= \textit{periodic payment} \\ i &= \textit{rate per period} \\ n &= \textit{number of payments (periods)} \end{aligned}$$

*Note that the payments are made at the end of each period.*

In the above formula, $i = \frac{r}{m}$, where $r$ is the interest rate (as a decimal) and $m$ is the number compounding periods per year and $n = mt$ where $t$ is the length of time of the investment. We can rewrite the formula with $r$ and $m$ instead of $i$

**Example 4.** *What is the value of an annuity at the end of* 10 *years if* \$1,000 *is deposited every* 3 *months into an account earning* 8% *compounded quarterly. How much of this value is interest?*

**Solution.**

**Example 5.** *If* \$1,000 *is deposited at the end of each year for* 5 *years into an ordinary annuity earning* 8.32% *compounded annually, what will be the value of the annuity at the end of the* 5 *years?*

**Solution.**

**Sinking Funds.** We can turn the annuities picture around and ask how much we would need to deposit into an account each period in order to get the desired final value. It is simple to solve for $PMT$ in the annuities formula to get

**Definition 2** (Sinking Funds)**.**

*where all the variables have the same meaning as for annuities.*

**Example 6.** *New parents are trying to save for their child's college and want to save up* \$80,000 *in* 17 *years. They have found an account that will pay* 8% *interest compounded quarterly. How much will they have to deposit every quarter in order to have a value of* \$80,000*?*

**Solution.**

**Example 7.** *A bond issue is approved for building a marina in a city. The city is required to make regular payments every* $3$ *months into a sinking fund paying* $5.4\%$ *compounded quarterly. At the end of* $10$ *years, the bond obligation will be retired with a cost of* $\$5,000,000$. *How much will the city have to pay each quarter?*

**Solution.**

# Section 3.4 - Present Value of an Annuity; Amortization

**Present Value of an Annuity.** In the next concept, we will look at making a large deposit in order to have a fund which we can make constant withdraws from. We make an initial deposit, then make withdraws at the end of each interest period. We should have a balance of \$0 at the end of the predetermined amount of time the fund should last.

**Example 8.** *How much should you deposit into an account paying* 6% *compounded semiannually in order to be able to withdraw* \$2000 *every* 6 *months for* 2 *years? (At the end of the* 2 *years, there should be a balance of* \$0 *in the account.)*

**Solution.**

This gives rise to the following formula

**Definition 3** (Present Value of an Ordinary Annuity).

*where*

$$\begin{aligned} PV &= \textit{present value} \\ PMT &= \textit{periodic payment} \\ i &= \textit{rate per period} \\ n &= \textit{number of payments (periods)} \end{aligned}$$

*Note that the payments are made at the end of each period.*

In the above formula, $i = \dfrac{r}{m}$, where $r$ is the interest rate (as a decimal) and $m$ is the number compounding periods per year and $n = mt$ where $t$ is the length of time of the annuity. We can rewrite the formula with $r$ and $m$ instead of $i$

**Example 9.** *How much should you deposit in an account paying* 8% *compounded quarterly in order to receive quarterly payments of* \$1,000 *for the next* 4 *years?*

**Solution.**